ECE 330 Exam #1, Fall 2018 Name: Solution
90 Minutes

Section (Check One) MWF 9 am ______ MWF 10 am ______

1. ________ / 25 2. ________ / 25

3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\quad \bar{V} = \bar{Z}\bar{I}$ $\quad \bar{S} = \bar{V}\bar{I}^* = P + jQ$ $\quad \bar{S}_{3\phi} = \sqrt{3} V_L I_L \angle\theta$

$0 < \theta < 180^\circ$ (lag)
$-180^\circ < \theta < 0$ (lead)

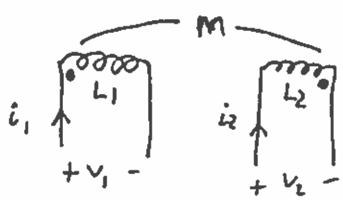
$I_L = \sqrt{3} I_\phi$ (delta)
$V_L = \sqrt{3} V_\phi$ (wye)

$\bar{Z}_Y = \bar{Z}_\Delta / 3$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

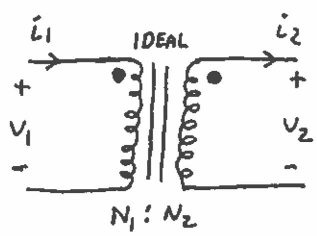
ABC sequence has A at zero, B at minus 120 degrees, and C at plus 120 degrees

$\int_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\quad \int_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\quad \Re = \frac{l}{\mu A}$ $\quad MMF = Ni = \phi\Re$

$\varphi = BA$ $\quad \lambda = N\varphi = Li$ (if linear) $\quad v = d\lambda/dt$ $\quad k = \frac{M}{\sqrt{L_1 L_2}}$ $\quad$ 1 hp = 746 Watts

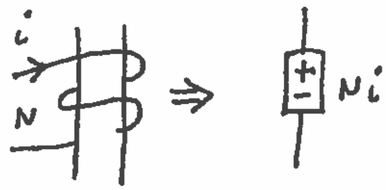


$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$



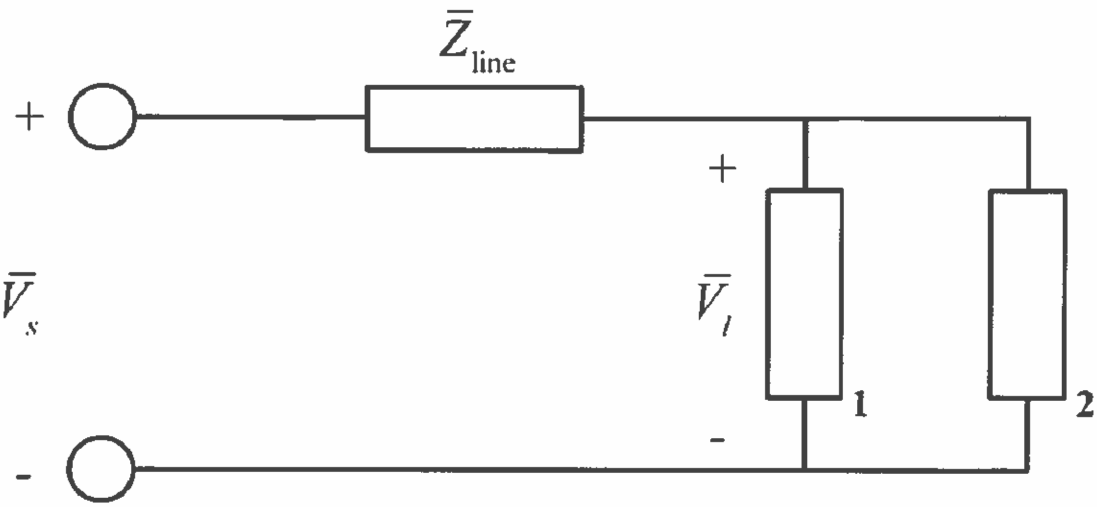
$a = \frac{N_1}{N_2}$ $\quad N_1 i_1 = N_2 i_2$

$\frac{v_1}{v_2} = \frac{N_1}{N_2}$



(extra paper at the end)

Problem 1:

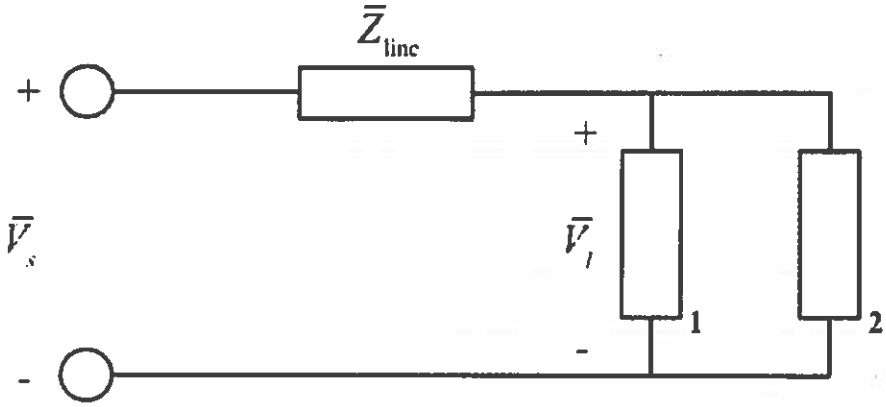


Two single-phase loads are connected in parallel to a voltage source through a feeder with impedance $\bar{Z}_{\text{line}} = 1 + j\sqrt{3}\ \Omega$. Load 1 consumes 1500 W of power at a power factor of 0.8 lagging. Load 2 consumes 1000 VA of power at a power factor of 0.6 lagging. The voltage at the loads is $v_l(t) = \sqrt{2}(120)\cos(377t)$ V.

Using the given values, find:

a) The total complex power consumed by both loads.
b) The total current supplied by the source.
c) The voltage as a function of time supplied by the source $v_s(t)$.
d) If a capacitor is connected in parallel with the two loads, what power must be supplied to achieve a total power factor of 0.9 lagging?

Problem 1 Solution



a) $P_1 = 1500\ W$ $\quad PF_1 = 0.8$ $\qquad\qquad S_2 = 1000\ VA$ $\quad PF_2 = 0.6$

$P_1 = S_1(PF_1) \Rightarrow S_1 = \frac{P_1}{PF_1} \Rightarrow S_1 = 1875\ VA$ $\qquad \bar{S}_2 = 600 + j800\ kVA$

$\bar{S}_1 = 1500 + j1125\ kVA$ $\qquad \bar{S}_{TOT} = \bar{S}_1 + \bar{S}_2 \Rightarrow$ $\boxed{\begin{aligned}\bar{S}_{tot} &= 2100 + j1925\ VA\\ &= 2848.8\angle 42.51°\ VA\end{aligned}}$

b) $\bar{V}_L = 120\angle 0°\ V$ $\qquad \bar{S}_{TOT} = \bar{V}_L \bar{I}_{tot}^*$

$\bar{I}_{tot} = \left(\frac{\bar{S}_{tot}}{\bar{V}_L}\right)^* \Rightarrow \boxed{\bar{I}_{tot} = 23.74\angle -42.51°\ A}$

c) $\bar{V}_s = \bar{Z}_{line}\bar{I}_{tot} + V_L \Rightarrow \bar{V}_s = (2\angle 60°)(23.74\angle -42.51°) + 120\angle 0°\ V$

$\bar{Z}_{line} = 2\angle 60°$ $\qquad \bar{V}_s = 47.48\angle 17.49° + 120\angle 0°$ $\Rightarrow \bar{V}_s = 165.285 + j14.27\ V$

$\bar{V}_s = 165.9\angle 4.93°\ V$

$\boxed{v_s(t) = \sqrt{2}(165.9)\cos(377t + 4.93°)}$

d) $Q_{tot} = 1925\ VAR \Rightarrow$ $\quad Q_C = -1925\ VAR$ or 1925 VAR of capacitance added.

Problem 2. (25 points)

A balanced, symmetrical, Wye-connected, three-phase load consumes a total of 1,000 Watts (3 phase) at a voltage of 208 V (line-line). The line current is 4 Amps and the power factor is lagging.

a) Find the **capacitance** needed for use in a Delta connection across the load to lower the line current to 3 Amps while the load still consumes the same real power. Assume a 60Hz supply.

$\bar{S}_0 = \sqrt{3}\times 208\times 4\angle\theta_0 = 1000 + jQ_0$ $\qquad \theta_0 = 46^\circ$

$\bar{S}_1 = \sqrt{3}\times 208\times 3\angle\theta_1 = 1000 + jQ_1$ $\qquad Q_0 = 1037$ vars

$\theta_1 = 22.3^\circ$

Need $1037 - 410 = 627$ VARs 3ϕ $\qquad Q_1 = 410$ vars

$627 = 3\times\dfrac{208^2}{X_C}$ $\qquad X_C = 207\,\Omega$ per phase $= \dfrac{1}{2\pi 60 C}$ $\qquad C = 12.8\,\mu F$

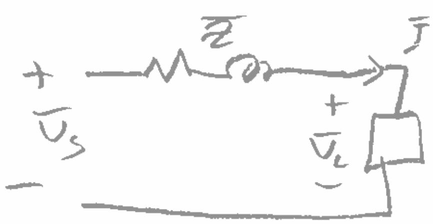
b) What would the line current be if these same capacitors were connected in a Wye rather than a Delta?

$Q_{new} = 3\times\dfrac{120^2}{207} = 209$ VARs 3ϕ

$\bar{S}_2 = \sqrt{3}\times 208\times I\angle\theta_2 = 1000 + j(1037-209)$

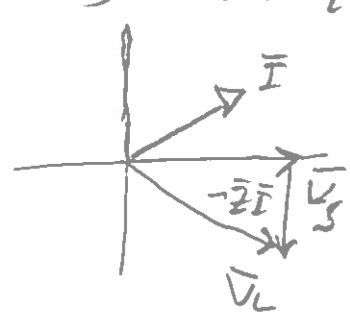
$\boxed{I = 3.6A}$ $\qquad = 1000 + j828 = 1298\angle 40^\circ$

c) In the real world, the line that serves the load has a series inductive impedance. If the source voltage is fixed, the load voltage will depend on the load power and the capacitors that are added. Show with a "per-phase" phasor diagram that if enough capacitance is added, the load voltage magnitude can be larger than the source voltage magnitude.



$\bar{V}_L = \bar{V}_s - \bar{Z}\bar{I}$

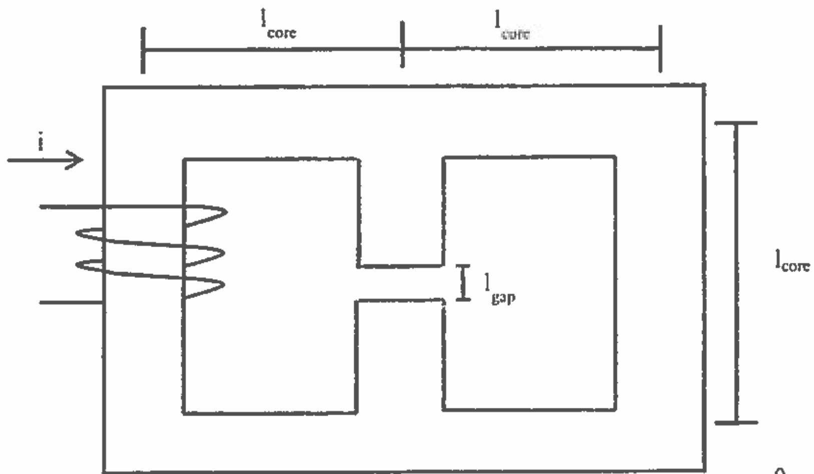
$\bar{V}_s = \bar{Z}\bar{I} + \bar{V}_L$



assume $\bar{Z}\bar{I} \approx v\angle 90^\circ$

Problem 3. (25 points)

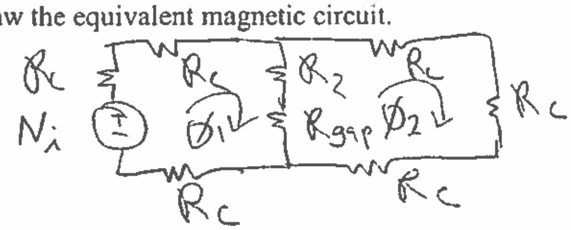
Consider the iron geometry given in the figure below. Assume fringing in the air gap such that $A_{gap} = 1.1*A_{core}$, and assume the following values: lcore = 10 cm, lgap = 0.1 cm, $A_{core} = 2\ cm^2$, N = 100, and $\mu_r = 1000$.



Sp 2013 exam

$$R_c = \frac{l}{\mu A} = \frac{0.1}{1000 \times 4\pi \times 10^{-7} \times 0.0002}$$
$$= 397887\ AT/W$$

(a) Draw the equivalent magnetic circuit.

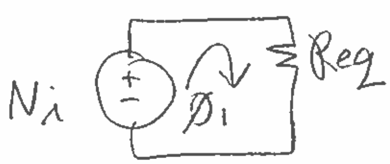


$$R_g = \frac{0.001}{4\pi \times 10^{-7} \times 0.00022} = 3.61716 \times 10^6\ AT/W$$

$$R_2 = \frac{(0.1 - 0.001)}{1000 \times 4\pi \times 10^{-7} \times 0.0002}$$
$$= 393908\ AT/W$$

(b) Find the inductance of the coil.

$$R_{eq} = 3R_c + [(R_2 + R_g) // 3R_c]$$
$$R_B = 919906\ AT/W$$
$$R_2 + R_g = 4.01107 \times 10^6\ AT/W$$
$$R_{eq} = 2.11 \times 10^6\ AT/W$$



$$Ø_1 = \frac{100i}{R_{eq}} \qquad L = \frac{NØ}{i} = \frac{N^2}{R_{eq}}$$
$$= 0.00473H$$
$$4.73\ mH$$

Continued on the next page

(c) Find the current needed to generate a flux in the middle leg of 5×10^{-6} Wb.

$$(\phi_1 - \phi_2) = 5\times10^{-6} \Rightarrow \phi_1 = 5\times10^{-6} + \phi_2 \qquad \phi_2 = \frac{MMF}{3R_c}$$

$$5\times10^{-6} \times (R_2 + R_g) = MMF = 20.0553$$

$$\phi_2 = 0.000017$$

$$\phi_1 = 5\times10^{-6} + 0.000017 = 0.000022$$

Loop 1: $-Ni + 3R_c\phi_1 + MMF = 0$

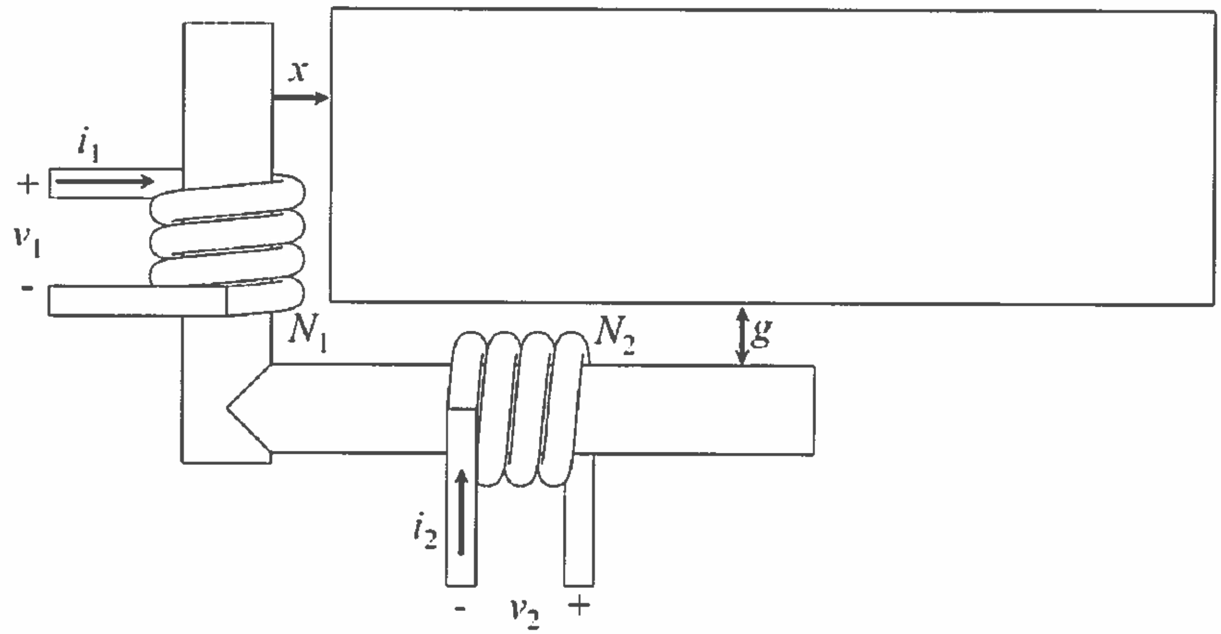
$$\underline{i} = (3R_c\phi_1 + 20.0553)/100 = \underline{0.463\,A}$$

(d) Find the flux density (Wb/m^2) in the right leg corresponding to the values given in part c.

$$\phi_2 = 0.000017$$

$$\underline{B_2} = \frac{\phi_2}{A_2} = \frac{0.000017}{0.0002} = \underline{0.085\ Wb/m^2}$$

Problem 4: (25 points)



One type of magnetic actuator consists of a moving piston and two coils, as shown above. Coil 1 acts as a constant MMF source with constant current i_1 whose direction is given. Coil 2 can produce an MMF that is used to either open or close the actuator through current i_2. The number of turns for each coil is given as N_1 and N_2 respectively. There is a constant air gap g between Coil 2 and the moving piston, whose position is given by x. The areas that the magnetic field acts through in the air gap and piston location are A_g and A_x respectively. Assume that the iron core and moving piston have infinite permeability, and that the magnetic flux acts all the way around a counter-clockwise loop.

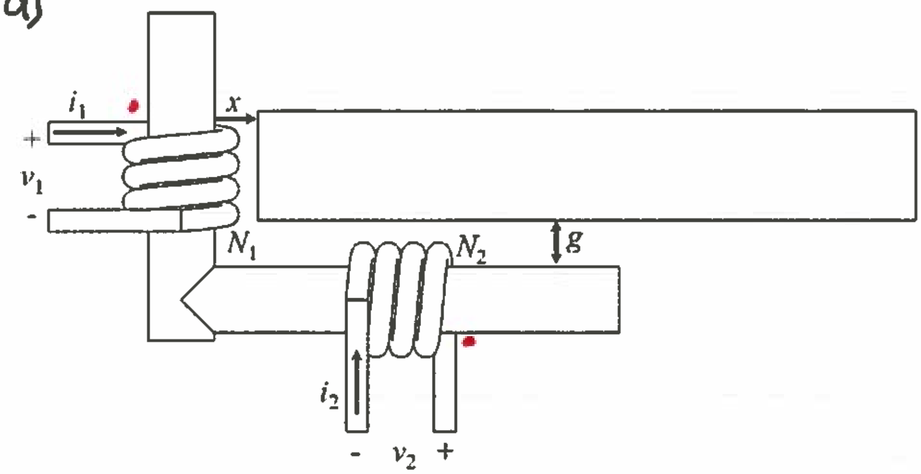
Using the current directions and polarity definitions given:

a) Find the dot convention for the given coils.
b) Draw the magnetic equivalent circuit for the actuator.
c) The self-inductance of coil 2, L_2, and the mutual inductance M in terms of x, g, N_1, N_2, A_g, and A_x.
d) Qualitatively, what happens to L_2 as the actuator opens (x increases from 0 to l): increase, decrease, or stay the same?
e) Find i_2 needed for zero flux through the iron.

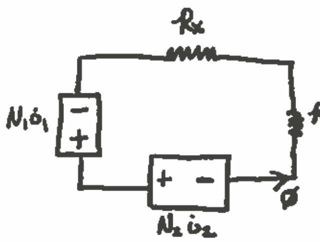
Extra paper on the next page

Problem ~~3~~ 4 Solution

a)



b)



c) $N_1 i_1 - N_2 i_2 = \phi (R_g + R_x)$

$N_1 i_1 - N_2 i_2 = \phi (R_g + R_x)$

$R_g = \frac{g}{\mu_0 A_g}$

$R_x = \frac{x}{\mu_0 A_x}$

$$\phi = \frac{N_1 i_1 - N_2 i_2}{R_g + R_x}$$

$$\phi = \frac{N_1 i_1 - N_2 i_2}{\frac{g}{\mu_0 A_g} + \frac{x}{\mu_0 A_x}}$$

$$\boxed{\lambda_2 = \frac{N_1 N_2 i_1 - N_2^2 i_2}{\frac{g}{\mu_0 A_g} + \frac{x}{\mu_0 A_x}} \Rightarrow \lambda_2 = \frac{\mu_0 A_g}{g + x\left(\frac{A_g}{A_x}\right)} \left[N_1 N_2 i_1 - N_2^2 i_2\right]}$$

$$\lambda_2 = M i_1 - L_2 i_2$$

$$\boxed{L_2 = \frac{N_2^2}{\frac{g}{\mu_0 A_g} + \frac{x}{\mu_0 A_x}} \Rightarrow L_2 = \frac{\mu_0 A_g N_2^2}{g + x\left(\frac{A_g}{A_x}\right)}}$$

d) As x increases, L_2 decreases

e) $\phi = \frac{N_1 i_1 - N_2 i_2}{R_x + R_g}$

$\phi = 0$

$N_1 i_1 - N_2 i_2 = 0 \Rightarrow \boxed{i_2 = \left(\frac{N_1}{N_2}\right) i_1}$